USN

Fourth Semester B.E. Degree Examination, December 2011

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define regular graph. Draw a graph which has 10 – vertices and 15 edges and which should be a 3 – regular graph. **(04 Marks)**

b. Define isomorphism of graphs. Determine whether the following graphs are isomorphic. **(06 Marks)**

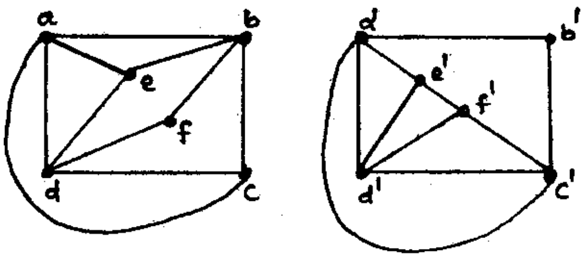

Fig. Q1(b)

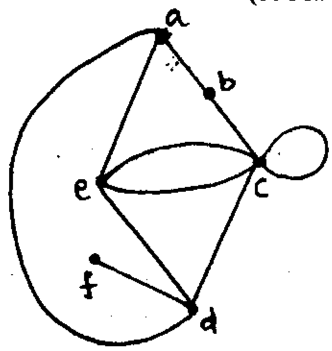

Fig. Q2(b)

c. If G is a graph with n – vertices and m – edges, let δ be the minimum and Δ be the maximal local degree, then prove that $\delta \le \frac{2m}{n} \le \Delta$. **(05 Marks)**

d. Give the comparison between Euler graph and Hamiltonian graph. **(05 Marks)**

2 a. Define complete bipartite graph. Prove that Kuratowski's second graph $K_{3,3}$ is nonplanar. **(07 Marks)**

b. Find the geometric dual of the following graph. **(07 Marks)**

c. Define chromatic number. Prove that a graph of order n(≥ 2) consisting of a single circuit is 2 – chromatic if n is even, and 3 – chromatic if n is odd. **(06 Marks)**

3 a. Define a tree. Prove that a tree with n –vertices has exactly (n – 1) edges. **(07 Marks)**

b. A tree has N_1 vertices of degree 1, N_2 vertices of degree 2, N_3 vertices of degree 3 and so on N_k vertices of degree k. prove that $N_1 = 2 + N_3 + 2N_4 + 3N_5 + \dots + (k-2)N_k$. **(06 Marks)**

c. Obtain an optimal prefix code for the message "FALL OF THE WALL". **(07 Marks)**

4 a. Find a minimal spanning tree for the following graph using KRUSKAL'S algorithm. **(07 Marks)**

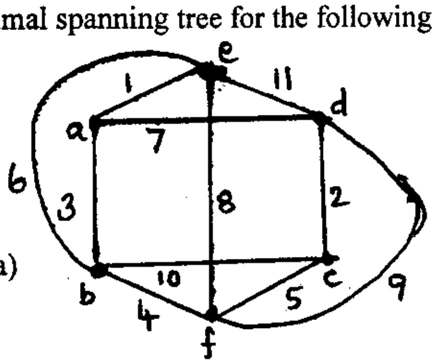

Fig. Q4(a)

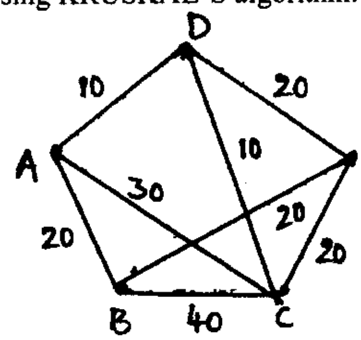

Fig. Q4(c)

b. Define matching, complete matching, cutest of a graph. Give one example for each. **(06 Marks)**

c. Explain the max–flow–min–cut theorem, apply this to network shown in Fig. Q4(c) to find the maximum flow possible between the vertices A and E. **(07 Marks)**

PART – B

5 a. Determine the number of 6 – digit integers (no leading 0's) in which
   i) No digit is repeated
   ii) No digit is repeated and it is even
   iii) No digit is repeated and it is divisible by 5. **(08 Marks)**

b. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(05 Marks)**

c. i) Show that $b_{n+1} = \dfrac{2(2n+1)}{(n+2)} b_n$, where b_n is the n^{th} Catalan number.
   ii) In how many ways can three semi–circles (can be of varying size) be drawn on a horizontal line so that the semi–circles do not intersect? Illustrate the various cases. **(07 Marks)**

6 a. In how many ways can 4a's, 3b's and 2c's be arranged so that all identical letters are not in a single block? **(07 Marks)**

b. Define derangement. In how many ways can the integers 1, 2, 3, 4, 5 be deranged? List those derangement where the first three numbers are 1, 2 and 3in some order. **(07 Marks)**

c. Obtain the rook polynomial for the following chessboard. **(06 Marks)**

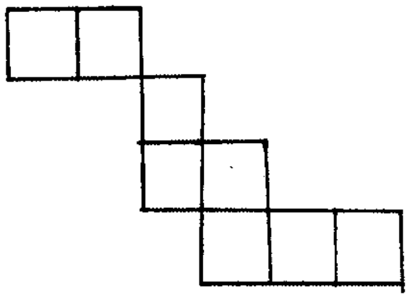

Fig. Q6(c)

7 a. In how-many ways can two dozen identical robots be assigned to four assembly lines with
   i) Atleast three robots assigned to each line
   ii) Atleast three but not more than nine robots assigned to each line. **(08 Marks)**

b. Show that the number of partitions of a positive integer n. where no summand appears more than twice equals the number of partitions of n, where no summand is divisible by 3. **(06 Marks)**

c. i) Find the exponential generating function for the sequence
   $0!, 1!, 2!, 3! \ldots$ and for $a, a^3, a^5, a^7, \ldots\ a > 0$
   ii) Determine the sequence generated by the exponential generating function $3e^{3x}$. **(06 Marks)**

8 a. Solve the recurrence relation
   $2a_n = 7a_{n-1} - 3z_{n-2}$, $n \geq 2$ and $a_0 = 2$, $a_1 = 5$. **(06 Marks)**

b. Solve the recurrence relation
   $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$
   where, $n \geq 0$ and $a_0 = 12$, $a_1 = 5$. **(08 Marks)**

c. By the method of generating function solve
   $a_{n+1} - a_n = 3^n$, $n \geq 0$, $a_0 = 1$. **(06 Marks)**

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